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International Journal of Solids and Structures 43 (2006) 1854–1855
INTERNATIONAL JOURNAL OF SOLIDS and STRUCTURES
www.elsevier.com/locate/ijsolstr

Discussion

Comment on "Stress boundary conditions for plate bending" by F.Y.M. Wan [Int. J. Solids Struct. 40 (2003) 4107–4123]

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Available online 25 August 2005

Abstract

By using the reciprocal theorem of elasticity, the author obtained the appropriate stress boundary conditions for the Levy solution for plate bending accurate to all order for plates of general edge geometry and loading. Two special cases of $k = 0$ (axisymmetric deformation of a circular plate) and $k \geqslant 2$ (unsymmetric deformation of a circular plate) were discussed in detail in the paper.

Keywords: Boundary conditions; Plate bending; Reciprocal theorem

However, for the first case, the author indicated, "*However, they determine only the Fourier coefficient b_0 but not the rigid body transverse displacement component a_0. Such a level of nonuniqueness is expected when all edge data are prescribed in terms of stresses.*" (p. 4115, lines 20–22). To determine the Fourier coefficient a_0, the second singular axisymmetric biharmonic function $w^{0'}(r) = r^2 \ln r$ in Eqs. (21) in Wan (2003) should be used. For this singular biharmonic function, the relevant displacement and stress components of the Levy solution are

$$\begin{aligned} U_r^* &= -\frac{z}{1-v}\left[(1-v)(2r\ln r + r) + \left(h^2 - \frac{2-v}{6}z^2\right)\frac{4}{r}\right], \\ U_\theta^* &= 0, \quad U_z^* = r^2 \ln r + \frac{2v}{1-v}z^2(\ln r + 1), \end{aligned} \tag{1}$$

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0020-7683/$ - see front matter
doi:10.1016/j.ijsolstr.2005.07.003

$$\Sigma^*_{rr} = -\frac{Ez}{1-v^2}\left[2\ln r + 3 + v(2\ln r + 1) - \left(h^2 - \frac{2-v}{6}z^2\right)\frac{4}{r^2}\right],$$
$$\Sigma^*_{r\theta} = 0, \quad \Sigma^*_{rz} = -\frac{2E}{1-v^2}(h^2 - z^2)\frac{1}{r}. \tag{2}$$

For the general non-singular axisymmetric biharmonic function $w_0 = a_0 + b_0 r^2$ in Eqs. (22) in Wan (2003), the relevant displacement and stress components of the Levy interior solution are

$$u^I_r = -2b_0 rz, \quad u^I_\theta = 0, \quad u^I_z = a_0 + b_0\left(r^2 + \frac{2v}{1-v}z^2\right), \tag{3}$$

$$\sigma^I_{rr} = -\frac{2E}{1-v}b_0 z, \quad \sigma^I_{r\theta} = \sigma^I_{rz} = 0. \tag{4}$$

Substituting expressions of displacement and stress Eqs. (1)–(4) into the reciprocal relation, one yields the following relation involved both a_0 and b_0:

$$\iint_E (\bar{\sigma}_{rr}U^S_r + \bar{\sigma}_{r\theta}U^S_\theta + \bar{\sigma}_{rz}U^S_z)_{*=k'c}\,\mathrm{d}S = 8\pi D\left[a_0 + \frac{4(4+v)}{5(1-v)}b_0 h^2\right]. \tag{5}$$

Being similar to the derivation in Section 6, according to Eq. (5) and Eq. (29) in Wan (2003), the rigid body transverse displacement component a_0 and the Fourier coefficient b_0 can be obtained. Hence, when the edge is subject to a prescribed set of admissible tractions, such a level of uniqueness is expected.